2.) Essentially, question can be defined as,

what is $P(R \mid P)$, where R is defined as "Rare",
And P is defined as "Has pattern"

$$\therefore P(R \mid P) = \frac{P(R \text{ and } P)}{P(P)} = \frac{0.001 \times 0.98}{([0.001 \times 0.98] + [0.999 \times 0.05])}$$

$$= \frac{(9.8 \times 10^{-4})}{0.05093} = 0.0192 \text{ (4dp)}.$$

20

Hence, there is a 1.924% ~~chance~~ the bug is of the rare species.

1.) ~~Ho:~~ Let the R.V "X" be defined as, "The # of people who ~~vote~~ vote for GIP in a sample of 10",
Hence, $X \sim B(10, p)$, where p is the probability that a Grainger votes for GIP.

$\Rightarrow$ $H_0 : p = 0.5$ ie, GIP do not hold majority.
$H_1 : p \neq 0.5$ ie, There is a majority.

2

Using a Hypothesis test, and assuming Ho is true, we can state,
if,
$P(X \geq 9 \mid H_0 \text{ true}) \leq 0.025$, then we can reject Ho.

$\therefore$ To find $P(X \geq 9 \mid H_0 \text{ true}) = P(X=9) + P(X=10)$
$= \binom{10}{9}(0.5)^9(0.5) + \binom{10}{10}(0.5)^{10}$
$= \frac{5}{512} + \frac{1}{1024} = \frac{11}{1024}$ or 0.0107 (4dp)

Hence, $0.0107 < 0.025$,

$\therefore$ Reject H_0.

Hence, using a two-tailed hypothesis test, at a 5% significance level, there is enough evidence to suggest ~~show~~ that H_0 is rejected, and rather that there is indeed a majority.

~~b.) Hence, let the R.V "Y" be defined as "The # of times 9 or more out of the 10 surveyed, in 5 attempts".~~

~~Hence, let the R.V "Y" be defin~~

~~Hence, the $P(X \geq 9 \mid H_0$ true$)$ in 5 attempts~~

~~$= \binom{5}{1}\left(\frac{11}{1024}\right)^1\left(1 - \frac{11}{1024}\right)^4$~~

~~$= 0.0514$ (4dp)~~

~~$\therefore$ Using if $P(X \geq 9 \mid H_0$ true~~

Using if $P(X \geq 9 \mid H_0)[$

q b.)

Hence, where the $P(X \geq 9 \mid H_0 \text{ true})$ happens at least once in 5 attempts

$= 1 - \left(1 - \frac{11}{1024}\right)^5$

$= 0.0526$ (4dp).

Hence, the actual probability of the party gaining 9 or more votes out of 10, at least once in 5 attempts $= 0.0526$.

$\therefore$ $0.0526 > 0.025$,

Hence H_0 is accepted.

$\therefore$ Hence, there isn't enough evidence, at a 5% significance level, to reject H_0

3d.) Using $\pi . P = \pi$, where π is a stationary matrix.

10p

$\rightarrow$ let $\pi = [s_1\ s_2\ s_3]$

$$\rightarrow [s_1\ s_2\ s_3]\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0.2 & 0 & 0.8 \end{bmatrix} = [s_1\ s_2\ s_3]$$

$\rightarrow 0.6s_1 + \cancel{0.4s_2}\ 0.2s_3 = s_1$ ①

$0.4s_1 + 0.6s_2 = s_2$ ②

$0.4s_2 + 0.8s_3 = s_3$ ③

where $s_1 + s_2 + s_3 = 1 \rightarrow s_1 = 1 - s_2 - s_3$ ④

Using ④ into ① gives,

$0.6(1 - s_2 - s_3) + 0.2s_3 - s_1 = 0.$

$\therefore 0.6 - 0.6s_2 - 0.6s_3 + 0.2s_3 - (1 - s_2 - s_3) = 0.$

$0.6 - 0.6s_2 - 0.4s_3 - 1 + s_2 + s_3 = 0.$

$0.4s_2 + 0.6s_3 = 0.4$

$\therefore 0.4s_2 = 0.4 - 0.6s_3$ ⑤

Using ⑤ into ③, gives,

$(0.4 - 0.6s_3) + 0.8s_3 = s_3$

$0.4 = 0.8s_3 \rightarrow s_3 = 0.5$

$\therefore s_2 = \dfrac{0.4 - 0.6(0.5)}{0.4} = 0.25$

$\therefore s_1 = 1 - (0.25) - (0.5)$

$= 0.25$

$\therefore \pi = [0.25\ 0.25\ 0.5]$

Hence, the employees are on the phone $1/4$ of the time.